

Exercise 35

Find the derivative. Simplify where possible.

$$G(t) = \sinh(\ln t)$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned} G'(t) &= \frac{d}{dt}[\sinh(\ln t)] \\ &= \cosh(\ln t) \cdot \frac{d}{dt}(\ln t) \\ &= \cosh(\ln t) \cdot \left(\frac{1}{t}\right) \\ &= \left(\frac{e^{\ln t} + e^{-\ln t}}{2}\right) \cdot \left(\frac{1}{t}\right) \\ &= \left(\frac{e^{\ln t} + e^{\ln t^{-1}}}{2}\right) \cdot \left(\frac{1}{t}\right) \\ &= \left(\frac{t + t^{-1}}{2}\right) \cdot \left(\frac{1}{t}\right) \\ &= \left(\frac{t + t^{-1}}{2} \times \frac{t}{t}\right) \cdot \left(\frac{1}{t}\right) \\ &= \left(\frac{t^2 + 1}{2t}\right) \cdot \left(\frac{1}{t}\right) \\ &= \frac{t^2 + 1}{2t^2} \end{aligned}$$